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AVA model in terms of softness parameter

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Abstract:
Taking the effect of variation of moment of inertia with angular momentum "concept of softness", and a harmonic variable moment of inertia model "AVM", a new formula is obtained, denoted AVNS. The AVNS model is used in calculating the energies of rotational ground bands of Sm^{150}, Sm^{152}, Gd^{154}, Os^{192}, Pt^{186}, Pt^{182} and Yb^{158} nuclei. The predicted results of the AVNS are in close agreements with AVM calculations and experimental data.

Keywords: Rotational bands; variable moment of inertia (VMI); angular momentum; softness parameter
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1. Introduction

From previous studies, it is confirmed that the Harris [1,2] two parameters ω^2 formula E= E= $\alpha\omega^2+\beta\omega^4$ is better than the two parameter I(I+1) expansion E(I)=AI(I+1) +B[I(I+1)]^2, where the first term represents the rotational part

$$"A=\frac{\hbar^2}{2\theta_o}$$

θ_o *is momen of inertia and I is nuclear spin follows the sequence* 0, 2, 4, 6 , "

And the second term represents the rotation vibration interaction, the two parameters A and B can be fitted from experimental data.

R.K. Gupta et al [3, 4]; proposed the deviation of observed data from two parameter **I (I+1**) expansion, may be attributed due to the change of moment of inertia. In this work, one merges the variation of moment of inertia and the concept of nuclear softness to formulate an equation "AVNS MODEL" which describing energies of states in rotation ground bands for Sm^{150}, Sm^{152}, Gd^{154}, Os^{192}, Pt^{186}, $Pt^{182}$and Yb^{158} nuclei. The predicted results of AVNS model are compared with experimental data and AVM model.

2. Method and Results

We know that the ground state bands of deformed nuclei are described by the formula [1, 2]

$$E(I)=\frac{\hbar^2}{2\theta_0}I(I+1) \qquad (1)$$

Introducing the effect of rotation – vibration interaction [3,4,5], we obtain:

$$E(I)=\frac{\hbar^2}{2\theta_0}I(I+1) \;+B[I(I+1)]^2 \qquad (2)$$

According to R. K. Gupta [6, 7, 8, 9] which introduce the concept of variation of moment of inertia with angular momentum i.e,

$$\theta(I)=\theta_0\left(1+\sigma_1 I+\sigma_2 I^2+\sigma_3 I^3+---+---\right) \qquad (3)$$

Where θ_0 is the moment of inertia at I=0, and σ_n is the softness parameter:

$$\sigma_n=\frac{1}{n!}\frac{\delta^n\theta(I)}{\delta I^n}\Big|_{I=0} \qquad (4)$$

$$n=1,2,3,\ldots$$

The anharmonic vibrator model (AVM) [5] is written as

$$E(I)=AI \;+\frac{I(I-2)}{\theta(I)}+\frac{1}{2}C\{\theta(I)-\theta_0\}^2 \qquad (5)$$

By using the softness concept up to first order, the previous Eq. (5) can be written in the form

$$E(I)=AI+\frac{I(I-2))}{\theta_0(1+\sigma_1 I)} \;+\frac{1}{2}C[\theta_0\sigma_1 I]^2 \qquad (6)$$

Using the experimental excitations energies $E(2)$, $E(4)$, $E(6)$ and $E(8)$ and the Equation (6), the parameters σ_1, θ_0, A and C: is written as

$$o_1 = \frac{\left[\frac{9E(2)-9E(4)+3E(2)}{3E(4)-E(2)+E(8)-3E(6)}-3\right]}{24} \quad (7)$$

$$\theta_0 = \frac{-48o_1}{[3E(2)-3E(4)+E(6)](1-4o_1)(1+6o_1)} \quad (8)$$

$$A = \frac{1}{24}\left[\frac{48}{\theta_0(1+480_1)}+16E(2)-E(8)\right] \quad (9)$$

$$C = \frac{1}{2}\left[\frac{E(2)-2A}{(\theta_0 o_1)^2}\right] \quad (10)$$

We calculated the energy levels of ground state bands for the chosen nuclei, the energies predicted according to Eq (6) "AVNS" model are compared with the experimental data and the results of AVM model [12] Eq. (5).

3. Results and Discussion:

Using the experimental excitation energies E (2), E (4), E (6) & E (8) and Equations (7, 8, 9 &10), the parameters σ_1, θ_0, A and C are calculated. Using Esq. (6) "AVNS" model and the given parameters in Table (1), we predicted the energies for chosen nuclei Sm150, Sm^{152}, Gd^{154}, Os192, Pt^{186}, $Pt^{182}$and Yb^{158}.which is listed in table (2). The deviation from experimental data is calculated as, $\tau = \frac{1}{N}\sum_{i=1}^{N}\left(E_{Cal} - E_{exp}\right)^2$.

The calculated results for the ground state rotational bands are given systematically in table 2 From this table we notice that the calculations are carried out for Sm^{150}, Sm^{152},Gd^{154},Os^{192},Pt^{186},Pt^{182} and Yb158 nuclei whose yrast bands are observed experimentally up to $I^{\pi} = 16^{+}$ for Sm^{150}, up to $I^{\pi} = 14^{+}$ for Sm^{152}, up to $I^{\pi} = 18^{+}$ for Gd^{154} and $I^{\pi} = 12^{+}$ up to for Os^{192} nuclei. And the energies calculated according to (AVNS) model in comparison with experimental data [13] and the energies calculated by AVM model for the chosen nuclei.in good agreement.

Table (1): The fitting parameters of AVNS model as in Eq. (5).

Nucleus	Parameters			
	σ_1	θ_0	A	C
^{154}Gd	4.877438E-02	58.10883	5.911697E-02	3.010189E-04
^{192}Os	6.802838E-02	52.23075	0.0908258	9.563694E-04
^{182}Pt	.0503732	82.19988	6.997559E-02	4.126185E-04
^{186}Pt	7.371714E-02	101.8626	8.413815E-02	2.059373E-04
^{150}Sm	.2606661	60.02434	0.156887	4.12082E-05
$^{!52}Sm$	6.818075E-02	67.09089	5.359747E-02	3.48998E-04
^{158}Yb	4.781804E-02	59.17679	0.1780455	1.441786E-04

From table (2), one clearly notices. That the predicted energies of AVNS model Eq (5), and AVM model Eq (5) are in close a with experiment as data.

Table (2): Experimental and Theoretical Energies in (Mev) of the Yrast band s of

I^{+}	EX	E(AVNS)	E(AVM)
2	0.2058	0.2058	0.20577
4	0.5803	0.5803	0.58028
6	1.0886	1.0886	1.0886
8	1.7081	1.7081	1.7081
10	2.4185	2.423519	2.4185
12	3.212	3.224157	3.212
Dev		2.88E-05	1.86E-10

^{182}Pt nucleus

I^+	EX	E(AVNS)	E(AVM)
2	0.1541	0.1541	0.1541
4	0.4175	0.4175	0.4175
6	0.7714	0.7714	0.7714
8	1.2024	1.2024	1.197
10	1.695	1.700691	1.681
12	2.238	2.258928	2.213
Dev		7.84E-05	0.000142

^{186}Pt nucleus

I^+	EX	E(VNS)	E(VAVM)
2	0.1915	0.1915	0.1915
4	0.4901	0.4901	0.4901
6	0.8772	0.8772	0.8772
8	1.3411	1.3411	1.334
10	1.8557	1.874072	1.848
12	2.407	2.470806	2.416
Dev		7.35E-04	3.18E-05

^{150}Sm nucleus

I^+	EX	E(VNS)	E(VAV)
2	0.33395	0.33395	0.33395
4	0.7735	0.7735	0.7735
6	1.27885	1.27885	1.27885
8	1.8371	1.8371	1.8371
10	2.432	2.442809	2.4174
12	3.043	3.093284	3.0339
14	3.646	3.78704	4.6758
16	4.306	4.523192	4.3401
Dev		8.71E-03	0.872045

^{152}Sm nucleus

I^+	EX	E(VNS)	E(VAV)
2	0.1218	0.1218	0.1218
4	0.3665	0.3665	0.3665
6	0.7069	0.7069	0.7069
8	1.1254	1.1254	1.1201
10	1.6093	1.610108	1.5912
12	2.1489	2.152699	2.1105
14	2.7363	2.747173	2.6714
Dev		1.90E-05	0.000863

^{158}Yb nucleus

I^+	EX	E(VNS)	E(VAV)
2	0.3584	0.3584	0.3584
4	0.8349	0.8349	0.8349
6	1.4042	1.4042	1.4042
8	2.048	2.048	2.044
10	2.7454	2.752737	2.741
12	3.4285	3.508143	3.485
Dev		1.07E-03	0.000538

^{154Gd} nucleus

I^+	Ex	E(VNS)	E(VAV)
2	0.1231	0.12307	0.12307
4	0.37101	0.37101	0.37101
6	0.71774	0.71774	0.71774
8	1.1445	1.1445	1.397
10	1.6372	1.63745	1.6219
12	2.185	2.186157	2.1543
14	2.778	2.782608	2.7299
16	3.4051	3.420543	3.346
Dev		3.263815E-05	0.00884241

The present study can also be useful in study the third term of equation (5) i.e. potential energy term with spin of the nucleons and with ground state of moment of inertia.

4. Conclusion

The present model Eq. (5) is practically fit to predict the ground state rotational bands of deformed even-even nuclei and can also be applied to nuclei where the energies of levels are experimentally available. It includes three parameters which are determined straight forward using linear least squares fitting.

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